

Assignment 4

Algorithms, Spring 2023

Honor code: *Work on this assignment alone or with one partner. Between different teams, collaboration is at level 1 [verbal collaboration only]. There are lots of resources online, such as animations, visualizations, practice problems, videos, and solutions— which you are encouraged to explore to deepen your understanding. However, you must be careful not to search for the specific problems in the assignment with the intent of getting hints for the solutions. Searching for the assignment problems on the internet violates academic honesty for this class.*

1. **Stoogesort:** One of your colleagues at work has proposed the following sorting algorithm, and your task is to evaluate it.

```
STOOGESORT( $A, i, j$ )
if  $A[i] > A[j]$ : swap  $A[i] \leftrightarrow A[j]$ 
if  $i + 1 \geq j$ : return
 $k \leftarrow \lfloor (j - i + 1) / 3 \rfloor$ 
STOOGESORT( $A, i, j - k$ )
STOOGESORT( $A, i + k, j$ )
STOOGESORT( $A, i, j - k$ )
```

(a) Correctness:

- do not turn in Work through an example and argue briefly that STOOGESORT correctly sorts any array of one element.
- do not turn in Work through an example and argue briefly that STOOGESORT correctly sorts any array of two elements.
- do not turn in Consider the algorithm but with the first line (that swaps elements $A[i]$ and $A[j]$) missing. Argue that it would not correctly sort by showing a simple counter-example.
- do not turn in Work through an example array of 3 elements and see how it is getting sorted by STOOGESORT.
 - i. Consider the state of the array A after the first recursive call finished and before starting the second recursive call (and assume the the recursive call correctly sorts). Consider the largest $n/3$ elements in A . Where might they reside? Make a claim and argue (briefly) why it's correct.
What we expect: Claim: Argument: ...

- ii. Consider the state of the array A after the second recursive call finished and before starting the third recursive call (and assume the recursive calls sort correctly). Consider the largest $n/3$ elements in A . Where might they reside? Make a claim and argue (briefly) why it's correct.

What we expect: Statement: Argument: ...

- (b) Running time: Give a recurrence for the worst-case running time of STOOGESORT and a tight asymptotic (Θ -notation) bound on the worst-case running time.

What we expect: The recurrence, illustrate the process to find its solution, and its solution.

2. (a) Warm-up problem: Let A and B be two sequences of n integers each. Given a target integer x , come up with an $O(n \lg n)$ algorithm for determining if there is an integer a in A and an integer b in B such that $x = a + b$. This is a warm-up problem, do not turn in your solution.

- (b) Consider now a generalization of the previous question: Given an array A of n integers, and a target integer x , describe an algorithm to return **true** if there exist three elements i, j, k (with $0 \leq i, j, k < n$ and $i \neq j \neq k$) in A whose sum $A[i] + A[j] + A[k]$ is x , or return **false** if no such subset exists.

We expect: (1) pseudocode and an English description of your algorithm; (2) analysis of its running time.

Evaluation

The assignment will be evaluated along several criteria:

1. **Correctness:** Is your solution correct?
2. **Justification:** Is your answer justified?
3. **Style:** Does it look professional and neat? Is the explanation written carefully in complete sentences, and well-organized logic? Is it easily human-readable? Is it easy to understand?
 - Assignments should be typed. Feel free to annotate the pdf to add figures and formulas which are too time-consuming to type.
 - Write each problem on a separate page or leave plenty of space between problems so that we can write comments.
 - Try to put yourself in the position of the reader. If you hadn't been thinking of this problem for 3 hours, would your answers make sense to you?
 - Try to finish the assignment early, then step away for a day or two, and then come back to it and read it again. Chances are you'll find something you can write more clearly.
 - Look at posted solutions for style advice (if solutions are not posted, ask).