Assignment 5

Algorithms, Spring 2023

Honor code: *Work on this assignment alone or with one partner. Between different teams, collaboration is at level 1 [verbal collaboration only]. There are lots of resources online, such as animations, visualizations, practice problems, videos, and solutions— which you are encouraged to explore to deepen your understanding. However, you must be careful not to search for the specific problems in the assignment with the intent of getting hints for the solution. Searching for the assignment problems on the internet violates academic honesty for this class.*

1. **Select the $\sqrt{n}$-closest:** Given an unordered sequence S of n elements (for simplicity, assume items are integers or real numbers), describe an efficient method for finding the $\lceil \sqrt{n} \rceil$ elements whose *values* are closest to (the value of) the median of S. What is the running time of your method? Aim for linear time.

   What we expect: The rationale of the algorithm, pseudocode, analysis.

2. **Merging sorted lists:** Assume you have k sorted arrays containing a total of n elements, and you want to merge them together in a single (sorted) array containing all n elements. For simplicity you may assume that the k arrays contain the same number of elements, namely n/k elements each.

   (a) Approach 1: merge array 1 with array 2, then merge the result with array 3, then merge the result with array 4, and so on. What is the worst-case running time ?

   What we expect: Detailed analysis of this approach

   (b) Approach 2: split the set of k arrays into two sets of $k/2$ arrays, merge each one recursively, then use the standard 2-way merge procedure (from mergesort) to combine the two resulting arrays. What is the worst-case running time ?

   What we expect: A recurrence , the recurrence depth, and the solution.

   (c) Approach 3: Give another approach (to merge the k arrays) that uses a heap, and runs in $O(n \lg k)$-time.

   What we expect: The idea of the algorithm, pseudocode and analysis.

Evaluation

The assignment will be evaluated along several criteria:

1. **Correctness**: Is your solution correct?

2. **Justification**: Is your answer justified?

3. **Style**: Does it look professional and neat? Is the explanation written carefully in complete sentences, and well-organized logic? Is it easily human-readable? Is it easy to understand?

   - Assignments should be typed. Feel free to annotate the pdf to add figures and formulas which are too time-consuming to type.
   - Write each problem on a separate page or leave plenty of space between problems so that we can write comments.
   - Try to put yourself in the position of the reader. If you hadn't been thinking of this problem for 3 hours, would your answers make sense to you?
   - Try to finish the assignment early, then step away for a day or two, and then come back to it and read it again. Chances are you'll find something you can write more clearly.
   - Look at posted solutions for style advice (if solutions are not posted, ask).